

AERODYNAMIC AND THERMAL CHARACTERISTICS OF A BODY OF  
COMPLEX SHAPE AROUND WHICH A HYPERSONIC AIR STREAM FLOWS

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The interaction between a sphere with a forward projecting spike and a dissociating hypersonic air flow is studied at high Reynolds numbers. The influence of the spike on the flow configuration, wave drag, and heat flux to the body surface is investigated.

As is known [1], a spike installed ahead of a blunt body permits a significant reduction in the wave drag coefficient at supersonic motion velocities. This governs the practical interest in bodies of such shape. A survey of early work is contained in [1]. Included are mainly the results of experiment that encompass a broad circle of questions associated with utilization of separation at the spike. The mentioned class of flows was investigated numerically in [2-4] within the framework of an inviscid formulation that permits determination of the flow configuration and the wave drag coefficient of the body. Solutions were obtained in a sufficiently broad range of variation of the body shape for low and moderate free-stream Mach numbers. At the same time, in addition to the aerodynamics, an analysis of the thermal characteristics of a streamlined body, particularly the heat flux to the surface and its equilibrium temperature, acquires important value in the study of such flows in the hypersonic velocity range ( $M_\infty \geq 6$ ) and at high Reynolds numbers. As experiments show [1, 5], the heat fluxes become so large in the mentioned range of parameters that they can cause thermochemical destruction of the body resulting in formation of recesses and caverns for which the earlier optimal aerodynamic characteristics change so much that the body can become unstable and its motion trajectory unpredictable. In this connection, both the aerodynamic and the thermal characteristics of a blunt body with a spike around a hypersonic air stream flow are investigated numerically in this paper for the class of flows under consideration. Such flows were studied earlier within the framework of the Navier-Stokes equations for low and moderate Reynolds numbers and  $M_\infty \lesssim 3$  in [6, 7].

Let us consider the hypersonic air flow around a spherical bluntness with a projecting spike and a hemispherical terminator (Fig. 1) at high Reynolds numbers. The presence of the small parameter ( $\epsilon = Re^{-1/2}$ ) in the problem permits its separation into two independent problems. The first is to compute the flow parameters in an inviscid shock layer and to determine the body wave drag. The second is to compute the flow in a laminar boundary layer on a permeable body surface possessing ideal catalytic properties and to determine the viscous drag, the heat flux to the surface, and its equilibrium temperature. The inviscid problem reduces to solving a system of Euler equations that is closed by the equation of state relative to the desired flow parameters. The numerical solution of these equations is obtained in the paper on the basis of the difference scheme of the method of large-scale particles [8]. To take account of the physicochemical processes in the hypersonic inviscid shock layer, the approximation [8] of the thermodynamic functions of equilibrium air, determining the dependences  $P = P(\rho, I)$  and  $T = T(\rho, I)$  in explicit form, is used in the algorithm. The wave drag coefficient was computed from the formula  $c_x = \int_0^1 p^* dr^2$ , where  $p^* = (P - P_\infty)/\rho_\infty V_\infty^2$ . The pressure distribution  $P(x)$  obtained on the outline of the body by solving the problem was approximated by orthogonal polynomials by least squares and was the input parameter for the solution of the second problem.

We now examine the question of determining the heat flux to an ideal catalytic permeable surface of an axisymmetric body around which dissociated air flows. As has been shown in [9, 10], in this case the magnitude of the heat flux referred to its value at the

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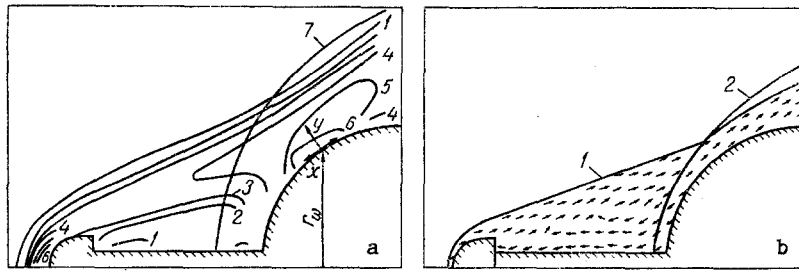


Fig. 1. Flow configuration around a body with a spike for  $l^0 = 0.75$ ;  $M_\infty = 15$ : a) lines  $\rho = \text{const}$  [1)  $\rho = 1.1$ ; 2) 1.3; 3) 1.5; 4) 2.0; 5) 3.0; 6) 4.0; 7)  $\rho = 1.1$ ;  $l^0 = 0$ ]; b) velocity field directions and shock wave shape [1, 2)  $l^0 = 0.75$ ; 0].

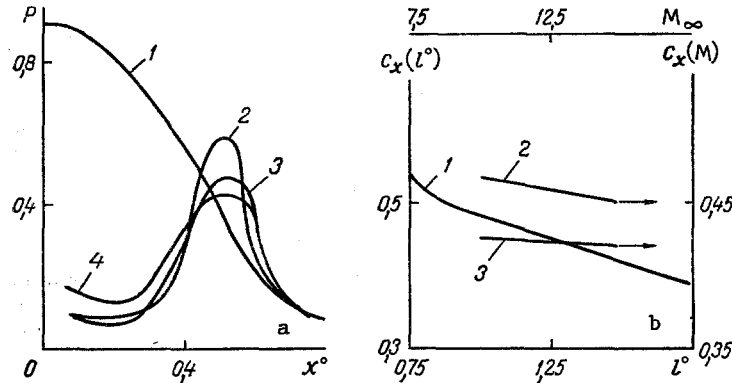


Fig. 2. Characteristics of inviscid flow around a body: a) dependence of the pressure  $P$  on the arc-length  $x^0$  [1)  $l^0 = 0$ ;  $M_\infty = 15$ ; 2) 1.5 and 15; 3) 1.5 and 10; 4) 0.75 and 15]; b) dependence of  $C_x$  on  $l^0$  [1)  $M_\infty = 15$ ] and on the Mach number [2, 3)  $l^0 = 1.25$ ; 1.5].

stagnation point  $q^0 = q_w(x)/q_w(0)$  depends weakly on the nature of the progress of the homogeneous chemical reactions in the gas and is determined with good accuracy by the value of this parameter evaluated for a homogeneous compressible gas flow around this same body. Meanwhile, in [11, 12] a formula was obtained to compute the heat flux at the stagnation point of an axisymmetric body with an ideal catalytic permeable surface

$$q_w(0) = a \frac{\sqrt{Re} \mu_0}{L} \sqrt{2\beta} (H_e - H_w) \left[ 1 + (Le^m - 1) \frac{h_d}{H_e - H_w} \right]. \quad (1)$$

The parameter  $m$  here depends on the mass flow rate of the gas through the surface [12], while the quantity  $a = (\theta'_{\eta} l) / (\text{Pr}(1 - \theta))_w$  is determined from the solution of the laminar boundary layer equations in a homogeneous gas.

Therefore, the heat flux to an ideal catalytic surface can be found from the formula

$$q_w(x) = q^0(x) q_w(0). \quad (2)$$

Here the quantity  $q_w(0)$  is determined from (1) and  $q^0(x)$  is found from the solution of the laminar boundary layer equations in a homogeneous gas which have the following form in the A. A. Dorodnitsyn variables [13]

$$\begin{aligned} (lu'_{\eta})'_{\eta} &= 2\xi(uu'_{\xi} - f'_{\xi}u'_{\eta}) - fu'_{\eta} + \beta(u^2 - \theta), \\ \left(\frac{l}{\text{Pr}}\theta'_{\eta}\right)'_{\eta} &= 2\xi(u\theta'_{\xi} - f'_{\xi}\theta'_{\eta}) - f\theta'_{\eta} - \alpha l(u'_{\eta})^2, \\ \xi &= \int_0^x \mu_e \rho_e u_e^* r_w^2 dx, \quad \eta = \frac{u_e^* r}{\sqrt{2\xi}} \int_0^y \rho dy, \quad l = \theta^{\alpha-1}, \end{aligned} \quad (3)$$

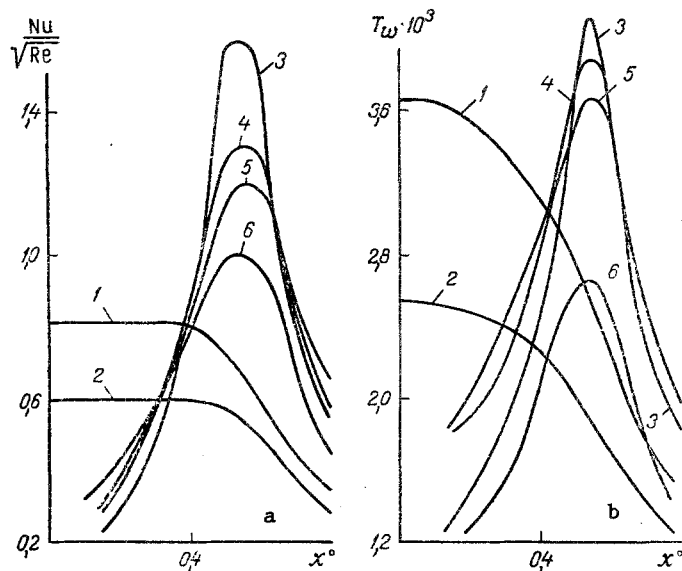


Fig. 3. Distribution of the heat transfer parameters  $Nu/\sqrt{Re}$  (a) and the equilibrium temperature of the surface  $T_w$ , °K (b) on an impermeable surface: 1)  $\lambda^\circ = 0$ ;  $M_\infty = 15$ ; 2) 0 and 10; 3, 4, 5)  $\lambda^\circ = 1.5$ ; 1.25; 0.75 for  $M_\infty = 15$ ; 6) 1.5 and 10.

$$u_e^* = (1 - P^{\frac{\gamma-1}{\gamma}})^{1/2}, \quad \alpha = \frac{2u_e^*}{T_e}, \quad \beta = \frac{2d \ln u_e^*}{d \ln \xi}.$$

The equations (3) were solved numerically on the basis of an implicit difference scheme of an elevated order of approximation [14] with the following boundary conditions

$$\begin{aligned} \eta = 0: \quad u = 0, \quad \theta = \theta_w, \quad \sqrt{2\xi} \frac{f}{v_w} = - \int_0^\xi \frac{(\rho v)_w d\xi}{\mu_e \rho_e u_e^* r_w}, \\ \eta = \infty: \quad u = \theta = 1. \end{aligned} \quad (4)$$

The case was also considered when the equilibrium temperature of the body surface was determined. The boundary condition  $\theta = \theta_w$  in (4) is here replaced by the condition

$$q_w(\xi) = \varepsilon \sigma T_w^4(\xi). \quad (5)$$

Profiles of the desired quantities across the boundary layer were determined during solution of the problem (3)-(5), as was also the heat transfer parameter

$$\frac{Nu}{\sqrt{Re}} = \frac{(Lc_p q)_w}{\lambda_w (H_e - H_w)} \frac{1}{\sqrt{Re}}.$$

The flow around a sphere and a sphere with a spike was studied for the following values of the governing parameters of the problem:  $\rho_\infty = 0.412 \text{ kg/m}^3$ ,  $T_\infty = 223.25^\circ\text{K}$ ,  $L = 0.2 \text{ m}$ ,  $M_\infty = 10-15$ ,  $\lambda^\circ = 0.75-1.5$ ,  $d = 0.2$ ,  $f_w = -2-0$ . Some results of the computations performed are shown in Figs. 1-4.

The influence of the spike on the configuration of the hypersonic flow around a sphere is shown in Fig. 1. It is seen that exactly as for low and moderate supersonic Mach numbers, the presence of the spike results in a qualitative change in the nature of the flow as compared with the flow around a sphere without a spike. A separation zone with circulation flow is formed between the body and the spike; domains with large local density gradients are formed in the neighborhood of the spike bluntness and the point of stream attachment; the shape of the shock changes to a significant extent.

The characteristic pressure distributions along the body surface are presented in Fig. 2a for different values of  $M_\infty$  and  $\lambda^\circ$ . The maximum pressure located at the point  $x^\circ = 0$  for  $\lambda^\circ \geq 0$  is shifted to the side surface of the sphere at the stream attachment point  $x^\circ \approx 0.5$  for  $\lambda^\circ \geq 0.75$ . The magnitude of this maximum is here reduced noticeably as compared

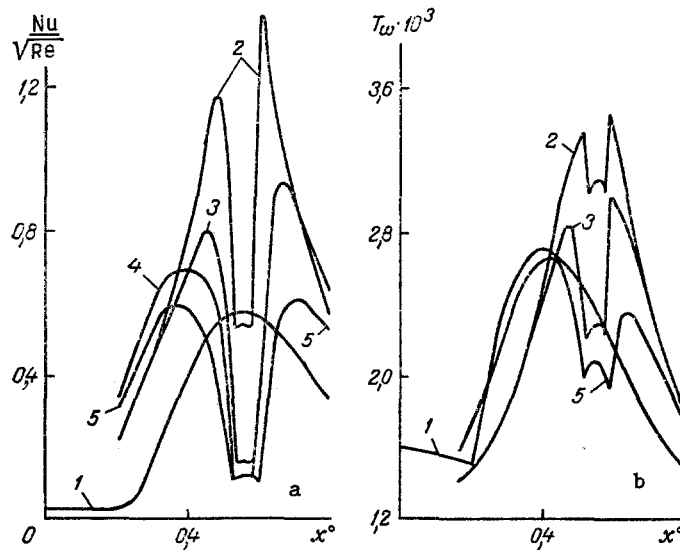


Fig. 4. Distribution of the heat transfer parameter  $Nu/\sqrt{Re}$  (a) and the equilibrium temperature  $T_w$ , °K (b) on a sphere in the presence of injection: 1)  $M_\infty = 15$ ,  $\ell^\circ = 0$ ,  $G = (\rho v)_w = 2.72$ ,  $x_b^\circ = 0.2$ ; 2, 3)  $G = 3.36$ ; 5.0 for  $\ell^\circ = 1.5$  and  $x_b^\circ = 0.055$ ; 4, 5)  $G = 3.39$ ;  $\ell^\circ = 0.75$ ,  $x_b^\circ = 0.055$  and 0.08.

with the case  $x^\circ \approx 0.5$ . Let us also note that although the dead zone grows with the increase in  $\ell^\circ = 0$ , the coordinate  $x^\circ$  of the point of attachment is practically independent of  $\ell^\circ$ .

Figure 2b on which the dependences of  $c_x$  and  $M_\infty$  on  $\ell^\circ$  are displayed yields a certain conception of the influence of the spike on the body wave drag coefficient. By comparison with a sphere for which  $c_x = 0.92$  in the Mach number range under consideration, the drag coefficient of a body with a spike is considerably smaller and is 0.57-0.4 depending on the length of the spike, i.e., is almost halved. For a fixed spike length an increase in the Mach number results in a small monotonic drop in  $c_x$ . At the same time for  $2 < M_\infty < 5$  the dependence  $c_x(M_\infty)$  is not monotonic in nature with a local maximum at  $M_\infty \approx 3$  [5].

We now examine the question of the influence of the Mach number and the body shape on its thermal characteristics. It is seen from Fig. 3 that, just as for the aerodynamic characteristics, the presence of the spike results in a qualitative change in the nature of these distributions. The maximums of  $T_w$  and  $Nu/\sqrt{Re}$  shift from the stagnation point in the neighborhood of the point of stream attachment and become more definite, while the very magnitude of these maximums for  $\ell^\circ = 0.75-1.5$  grows as  $\ell^\circ$  and  $M_\infty$  increase. The Mach number has the greatest influence on the value of the surface equilibrium temperature. Thus an increase in  $M_\infty$  from 10 to 15 results in an almost 40% growth in  $T_w$  while the change in  $T_w$  for  $\ell^\circ = 0.75$  and  $\ell^\circ = 1.5$  is not more than 10%, other things being equal. The behavior of the heat flux along a sphere, as obtained in this paper, is qualitatively in agreement with the Crawford experimental data [1].

Also examined was the case when the body surface had a permeable section through which gas was injected uniformly at the constant flow rate of  $(\rho v)_w = G = \text{const} < 5$ . For flow around a sphere without a spike the injection was in the neighborhood of the stagnation point at the arc length  $x_b^\circ = 0.2$  while in the presence of a spike the center of the permeable section was at the point of stream attachment and had the length  $x_b^\circ = 0.055$  and 0.08. Let us note that for  $G = O(1)$  the specific impulse of the gas being injected is much less than the specific impulse of the free stream. As analysis shows [15], in this case the influence of injection on the external inviscid flow characteristic can be neglected in a first approximation, and the pressure distribution obtained for the case of the flow around a body of the same shape with a nonpermeable surface can be used to compute the flow in the boundary layer.

The influence of injection on the heat transfer intensity is shown in Fig. 4. Injection of a gas results in a significant diminution in the heat flux and temperature of the surface.

Minimums of  $Nu/\sqrt{Re}$  and  $T_w$  are located here on the permeable surface section, and local maximums of these quantities in the neighborhood of points of cessation of injection. It is important to note that values of these maximums are substantially less than the corresponding maximums for the flow around a body without injection. Computations showed that for a fixed Mach number the surface equilibrium temperature depends strongly on the value  $f_w$  of the injection parameter. For instance, as it increases from 1.2 to 2.0 the surface temperature diminishes by 20-40% at corresponding points. Let us also note that a diminution in the spike length (for  $\ell^\circ = 0.75-1.5$ ) results in an increase in the efficiency of injection, other conditions being equal.

#### NOTATION

M, Re, Pr, Nu, Le, criteria; x, y, coordinates along the body contour and along its normal; u, v, velocity components in the x and y directions; P, pressure;  $\rho$ , density; T, temperature;  $\mu$ , viscosity;  $\omega:\mu = T^\omega$ ; L, body diameter; r,  $\ell^\circ$ , d,  $x^\circ_b$ , body radius, spike length, diameter of spike bluntness, length of the injection section referred to L; I, specific internal energy;  $c_x$ , wave drag coefficient; q, heat flux; H, total enthalpy;  $h_d$ , dissociation enthalpy;  $\theta = T/T_e$ ;  $\gamma$ , effective value of the adiabatic index; f, stream function;  $\epsilon$ , body integral emissivity;  $\sigma$ , Boltzmann constant; and  $\lambda$ , heat-conduction coefficient. Subscripts: 0, stagnation point;  $\infty$ , unperturbed stream; w, body surface; e, outer boundary layer boundary.

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